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Engineering Notes

ENGINEERING NOTES are short manuscripts describing new developments or important results of a preliminary nature. These Notes cannot exceed 6 manuscript pages and 3 figures; a page of text may be substituted for a figure and vice versa. After informal review by the editors, they may be published within a few months of the date of receipt. Style requirements are the same as for regular contributions (see inside back cover).

Maximum Power Absorption with Active Struts

Rakel Kristin Kanestrøm*
Stord/Haugesund College, Haugesund, N-5500 Norway
and
Olav Egeland†
Norwegian Institute of Technology, Trondheim, N-7034 Norway

Introduction

THE topic considered is vibration suppression in flexible structures by using active struts. From electrical circuit theory1 we know that for a network operating in the sinusoidal steady state, maximum average power is delivered to a load when the load impedance is the conjugate of the impedance of the network as viewed from the terminals of the load. By analogy, Chen and Lurie[2] proposed that the active strut impedance should be adjusted with respect to the load impedance facing the active member to maximize the vibration damping of the load. To ensure stability, the strut impedance was restricted to be positive real, and a constant real impedance equal to the average of the load impedance modulus over the whole frequency range was chosen. This impedance was realized using bridge feedback from velocity and force.

In this Note we argue that the strut impedance should be matched to the load impedance at the resonance peaks rather than to the average of the load impedance modulus over the whole frequency range. As the vibrational energy is concentrated around the resonant peaks, it makes sense that priority should be given to maximize the vibration suppression at the resonant frequencies. This approach does not rely on knowledge of the structural resonant frequencies. Furthermore we discuss the use of the desired active strut impedance. In this case the force feedback approach is found most attractive.

Impedance Matching

Consider an active strut consisting of a piezoelectric actuator in series with a piezoelectric force sensor. The strut may be modeled as a nominal length ℓ_0 and extension d in series with a lumped spring of stiffness K_e (Fig. 1). Here d is considered to be the control variable. Assume that the mass of the strut is negligible, and hence the force on both terminals are equal, and given by

$$f_1(s) = f_2(s) = f(s) = \frac{K_e}{s}[v_1(s) - v_2(s)] + K_e d(s) \tag{1}$$

where v_1 and v_2 are the endpoint velocities. Assume that f is measured and that the control law is $d(s) = -G(s)f(s)$. Inserting the control law into Eq. (1) and rearranging, we get $v(s) = [s/K_e + sG(s)]f(s)$, where $v = v_1 - v_2$. Following Chen and Lurie[2] we define the mechanical impedance Z by associating velocity v with voltage and force f with current, giving $v(s) = Z(s)f(s)$. The mechanical impedance $Z(s)$ of the active strut is hence given by

$$Z(s) = \frac{v(s)}{f(s)} = \frac{s}{K_e} + sG(s) = Z_0(s) + Z_g(s) \tag{2}$$

where $Z_0(s)$ is the inherent open-loop mechanical spring impedance $Z_0(s) = s/K_e$. Closing the force feedback loop hence corresponds to the insertion of an impedance $Z_g(s) = sG(s)$ in series with the open-loop spring impedance. Using the transfer matrix from actuator extension to force sensor measurement,[3] the impedance of a flexible truss structure $Z_s(j\omega)$ as seen from the interface between the active member and the structure is found to be

$$Z_s(j\omega) = \sum_{k=1}^{\infty} Z_{s,k}(j\omega) = \sum_{k=1}^{\infty} \frac{j\omega\left(-\omega^2 + \omega_k^2\right) + 2\zeta_k\omega_k\omega^2}{\left(-\omega^2 + \omega_k^2\right)^2 + (2\zeta_k\omega_k)^2}\nu_k \tag{3}$$

where ν_k depends on the location of the strut and is a measure of how good the kth vibration mode is observed by the strut.[4] Here, ω_k is the kth resonant frequency. It is observed that when $\omega = \omega_k$ the imaginary part of the kth term of $Z_s(j\omega)$ disappears and the real part reads $\mathrm{Re}[Z_{s,k}(j\omega_k)] = \nu_k/2\zeta_k\omega_k$.

In large flexible space structures the relative damping factors ζ_k are supposed to be very small, resulting in the high resonant peaks. At a resonance ω_k, the kth term makes the largest contribution to the impedance magnitude $|Z_s(j\omega_k)|$, and the approximation $Z_s(j\omega_k) \approx Z_{s,k}(j\omega_k)$ is justified. It is known that it is the resonances that cause the main difficulties with flexible structures. The main goal should hence be to absorb the maximum amount of power at the resonances. Due to the fact that the structural impedance is purely real at the resonances, an exact impedance match at these frequencies is achieved by making the strut impedances real and equal to the structural impedance magnitudes at the resonances, i.e., $Z_{\mathrm{opt}}(j\omega_k) = Z_s^*(j\omega_k) = \nu_k/2\zeta_k\omega_k$, where ω_k is a resonant frequency. Based on this, we propose to make the inserted strut impedance $Z_g(j\omega)$ real for all frequencies within the bandwidth of the equipment used for implementation. The magnitude should be chosen equal to the magnitude of the structural impedance at the resonant frequency that the strut is to suppress. As the magnitude of all the resonant peaks are high, the strut will provide good impedance match for all resonant frequencies. The performance will not be sensitive to errors in the natural frequencies because the structural impedance will be real at all resonance frequencies. As the structure will not attain large vibrational motion with frequencies that lie between two adjacent resonant frequencies, a poorer impedance match for these frequencies is supposed not to cause any trouble.

A purely real $Z_g(j\omega)$ describes the impedance of a dashpot and may be written $Z_g(j\omega) = 1/D$, where D is the dashpot constant. From Eq. (2), the controller that realized the dashpot impedance is found to be $G(s) = Z_g(s)/s = 1/Ds$. This equals the integral force

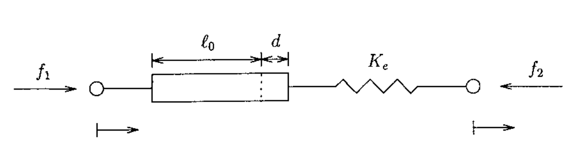

Fig. 1 Model of active strut.

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*Associate Professor, Department of Engineering.
†Professor, Department of Engineering Cybernetics. Associate Member AIAA.

feedback scheme proposed by Preumont et al.[4] and has been shown to be L_2 stable for an arbitrary number of active struts.[3]

Kanestrøm and Egeland[5] provided an experimental investigation of impedance-adjusted active struts. A purely real impedance realization showed significantly better vibration attenuation than a parallel spring-mass-dashpot configuration, even if this could be tuned to provide better magnitude match with the structural impedance for frequencies adjacent to the resonant frequency. This dynamic impedance could however only be tuned to one resonance, and the other resonances were not affected. The real impedance, however, resulted in attenuation of all the resonances that were observed by the active strut. These results support our proposed approach.

Force Feedback Versus Bride Feedback

Now consider the bridge controller[2] given by $d(s) = -C(s)[Z'(s)f(s) - v(s)]$, where $Z'(s)$ is the mechanical impedance to be realized by the active strut and $C(s)$ is the compensator. Combining this equation with Eq. (1) gives

$$Z(s) = \frac{v(s)}{f(s)} = Z_0(s)\frac{1 + sC(s)Z'(s)/Z_0(s)}{1 + sC(s)} \tag{4}$$

If $|C(s)|$ is sufficiently large, Eq. (4) can be approximated by $Z(s) \approx Z'(s)$. The advantage of this technique is that for large $|C(s)|$ the resulting impedance is independently of the inherent impedance $Z_0(s)$ of the strut. This gives insensitivity to parameter uncertainty in $Z_0(s)$. The bridge feedback solution has the advantage of having a high in the feedback controller even if the specified impedance to be realized is small in magnitude, so that disturbances and imperfections in the actuator will be attenuated. However, it was reported by Chen and Lurie[2] that the required magnitude of $C(s)$ was not achieved, and hence there were problems in realizing $Z(s) = Z'(s)$.

In our scheme we arrive at $v(s) = [Z_0(s) + sG(s)]f(s)$. Here, $Z_0(s)$ appears in the resulting impedance $Z(s) = Z_0(s) + sG(s)$ of the strut. This it not a problem since $Z_0(s)$ is the inherent mechanical spring impedance of the open-loop strut and will hence not destabilize the system. As long as the impedance component $sG(s)$ in series with $Z_0(s)$ is strictly passive with finite gain, L_2 stability is ensured regardless of the parameter in $Z_0(s)$.[3,6] The controller $G(s)$ that realizes the impedance $Z(s) = Z_0(s) + sG(s) = Z'(s)$ is given by $G(s) = [Z'(s) - Z_0(s)]/s = Z'(s)/s - 1/K_e$. The inherent axial stiffness K_e is assumed to be large such that $K_e \gg 1$. The controller is hence dominated by the first term and may be written $G(s) \approx Z'(s)/s$ and equivalently $Z(s) \approx Z'(s)$. Hence, there will be no problems in realizing a real impedance. For the flexible structure the required impedance is so high that it in fact is unattainable and $sG(s)$ has to be tuned to the maximum value that gives stability in the presence of measurement noise and discretization and sampling effects. There will hence be a high gain in the feedback controller, and the required disturbance attenuation is obtained. A great advantage with the force feedback approach is that it requires only one requirement and avoids the use of differentiation in the control scheme.

Conclusions

Active struts are shown to provide best vibration suppression when the closed-loop strut impedance is made real and with high magnitude. In particular, the performance does not depend on knowledge of the structure's natural frequencies. The ability to suppress a specific resonance depends on where the strut is located in the structure and the limit on the controller gain. One strut absorbs power due to all the resonances that are observable at its location. The only uncertainty is the structural damping, which influences the height of the resonant peaks and hence defines the magnitude of the best controller gain. The magnitude of the structural impedance is supposed to be large at the resonant frequencies. Probably, the active strut hardware will restrict the impedance magnitude of the strut. The tuning procedure will hence be to choose the highest impedance that does not destabilize the system in the presence of measurement noise and phase lags due to a digital implementation of the controller.

A purely real, positive active strut impedance corresponds to the insertion of a dashpoint in series with the open-loop passive strut. This is realized using an integral force feedback scheme. The scheme is L_2 stable for an arbitrary number of active struts and is attractive since it avoids the use of differentiation.

Acknowledgments

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Two Misconceptions in the Theory of Inertial Navigation Systems

Itzhack Y. Bar-Itzhack*
Technion—Israel Institute of Technology, Haifa 32000, Israel

I. Introduction

THE purpose of this Note is to point out prevailing wrong explanations of two basic facts in the operation of inertial navigation systems (INSs) and provide the correct explanation for them.

II. Gyrocompassing

A. Misconception

Principle of Operation

For simplicity let us consider the gyrocompassing process of a stable platform INS. Denote Earth rate by Ω and latitude by λ; then it is well known that Ω_N, the local north component of Earth rate at a point on Earth whose latitude angle is λ, is given by $\Omega_N = \Omega \cos \lambda$. Due to azimuth misalignment, the platform north and east coordinate axes, N_p and E_p, respectively (see Fig. 1), are rotated by the azimuth misalignment angle ψ_D with respect to N and E, the reference, local level local north (LLLN) coordinate system. The projection of Ω_N on the platform coordinate axes yields the component $-\Omega_N \sin \psi_D$ along the E_p axis that, for a small ψ_D, becomes $-\Omega_N \psi_D$. The east platform gyroscope, which measures rates along the E_p axis, measures the component $-\Omega_N \psi_D$, which is treated as a platform drift error and is fed into the platform control system. The latter, in an attempt to cancel the unwanted drift, torques the platform in an opposite direction and thus causes the platform to drift about its east axis by the rate $\Omega_N \psi_D$. To this drift we add

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*Sophie and William Shamban Professor of Aeronautics, Faculty of Aerospace Engineering; Head, Flight Control Laboratory; Member, Technion Space Research Institute.